The Spring-Mass Oscillator

Goals and Introduction

In this experiment, we will examine and quantify the behavior of the spring-mass oscillator. The spring-mass oscillator consists of an object that is free to oscillate up and down, suspended from a spring (Figure 19.1). The periodic motion of the object attached to the spring is an example of harmonic motion – a motion for which the acceleration is always directed oppositely from the displacement of the object from an equilibrium position.

When an object with mass m is hung from a spring with spring constant k, the spring stretches, changing its length by an amount x. When motionless, the spring-mass system is in equilibrium. There is a gravitational force pulling down on the mass and the spring restoring force pulling up on the mass. The spring restoring force is given by

$$F_{\text{spring}} = -kx, \quad \text{(Eq. 1)}$$

where the k is the spring constant in units of N/m and x is the extension or compression of the spring from its natural length. The displacement, x, could be positive or negative depending on whether the spring is compressed or stretched (we would need to decide the direction of the positive x-axis). The minus sign in Eq. 1 indicates that the direction of the spring restoring force always opposes the direction of the displacement from the equilibrium position. We can say in general, however, that when the spring-mass system is in equilibrium, $\left|F_{\text{spring}}\right| = \left|F_{\text{gravity}}\right|$, or $kx = mg$.

In Figure 19.1, we see an example of a spring-mass system where the equilibrium position above the location of a detector is noted. It is displacement from this equilibrium position that will then cause the system to oscillate. If the object, or mass, is pulled downwards a distance A from the equilibrium position and then released, the spring restoring force will initially cause the object to accelerate upwards. This would continue until the object moves above the equilibrium position, and the spring compresses past that point. The spring is then pushing downwards on the object to try to get it back to the equilibrium position, and it begins to slow down. You might say that when the object is displaced from the equilibrium position and released, it is always being pushed or pulled by the spring in an effort to return it to the equilibrium position.

In simple harmonic motion, the displacement of the object from the equilibrium position will behave sinusoidally. This means that when we graph the position of the object over time while it oscillates, we should see a curve that is similar to a sine or cosine function. This is also true for the velocity and the acceleration of the object over time. If the positive x-axis points upwards in

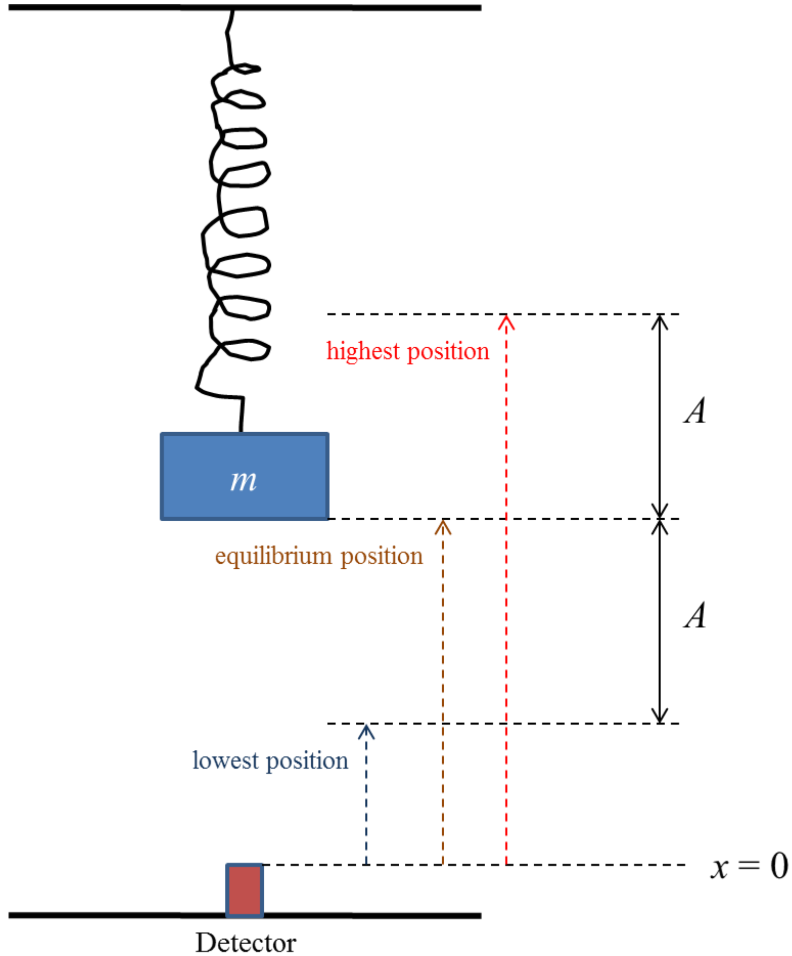

Figure 19.1

our picture, the position of the object will first have a value less than the equilibrium position, begin to increase, reach some maximum value a distance *A* above the equilibrium position, and then decrease until it returns to the release point, a distance *A* below the equilibrium position. The motion is symmetric, as indicated in Figure 19.1.

One can find a similar oscillatory behavior for the velocity and acceleration, but they are not in sync with each other or with the position as a function of time. In other words, just because the position is increasing and "positive" (above the equilibrium position) does not mean that the velocity is also increasing and positive (above a velocity of 0).

There are some expected features of simple harmonic motion for the spring-mass system that we should verify in any data set before proceeding with further analysis. A detector will be placed below the spring-mass system and will be used to collect data on the position, velocity and acceleration of the mass as a function of time, while it is oscillating. The data will be displayed as three graphs and the following behaviors should be observed in these graphs:

1) When the object reaches a maximum position (either above or below the equilibrium point), the velocity should be 0 at that instant.

2) When the object reaches a maximum position (either above or below the equilibrium point), the acceleration should be at an extreme. In other words, the acceleration should be at its maximum positive or maximum negative value (depends on the direction of the spring restoring force at that instant)

3) When the object is at the equilibrium position (moving through it), the velocity should be at a maximum. In other words, the velocity should be at its maximum positive or maximum negative value (depends on whether it is moving up or down at that instant)

4) When the object is at the equilibrium position (moving through it), the acceleration should be 0 at that instant. This is because the spring is back to a length where its restoring force is equal to the gravitational force on the object.

It is also worth noting that once the spring-mass system is set into motion, we expect that the total mechanical energy, *E*, of the system should be conserved. This is because the spring restoring force is a conservative force, like the gravitational force. For small oscillations, we can ignore the gravitational potential energy and approximate the total energy in the spring-mass system as

$$E = KE + PE_{spring} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \quad \text{(Eq. 2)}$$

where x is the amount of compression or stretch of the spring <u>measured from the equilibrium position</u>. This means that during the motion, x will never be bigger than A, the amplitude of the motion.

Because the total mechanical energy should be conserved, it should be the case that if we calculate E at different moments in time, it should be the same.

Another interesting aspect of this simple harmonic motion can be found by further examining the relationship between the position and acceleration as functions of time. The time it takes the spring-mass to go through one complete oscillation (from one extreme position to the other, and then back to the starting extreme) is called the period, T. Therefore, the period can be found from the *position* vs. *time* (or x vs. t) graph. If we look at the amount of time that has passed from one peak to the next on the plot (remember it will look like a sine function), this should be equal to the period! The object is leaving a position and arriving there again, moving in the same direction, at a later time; one cycle has been completed.

An event that is periodic may also be described in terms of its frequency, *f*, or how many times the oscillation repeats per second. The period and frequency of an oscillation are related:

$$f = \frac{1}{T} \quad \text{(Eq. 3)}$$

Careful analysis suggests that the period, and thus the frequency, is dependent upon the spring constant, *k*, and the mass of the object, *m*. The prediction is that the frequency for the simple harmonic motion of a spring-mass system should be given by

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \text{(Eq. 4)}$$

Note that this frequency is independent of the amplitude of the motion!

Here, we intend to measure the period of the spring-mass system, the spring constant, and the mass of the object in an effort to confirm the validity of the relationship in Eq. 4. Along the way, we also hope to verify the predicted sinusoidal behavior of the three kinematic quantities (position, velocity, and acceleration) and investigate the conservation of energy that should be evident during the motion.

<u>*Goals*</u>: (1) Measure and consider aspects of the spring-mass oscillator.
(2) Test the validity of the Eq. 4 by measuring the period, spring constant, and mass.
(3) Verify the sinusoidal behavior of the kinematic quantities of the spring-mass oscillator.
(4) Verify the conservation of energy during the motion of the oscillator.

Procedure

Equipment – spring, mass holder with removable masses, meter stick (or other distance-measurement tool), balance, motion detector, computer with the DataLogger interface and LoggerPro software

The basic setup should be completed for you prior to lab, as shown in Figure 19.1. We will need to calibrate this using the following steps (if there is no setup, your TA should aid the class in getting to this calibration point). The motion detector should be on the floor with a protective shield over it. Above the detector, the mass holder will hang from the spring.

1) **Measure and record** the mass of the mass holder, using the balance. Label this as m_{holder}.

2) If it is not already done, hang the spring from the support that should be setup for you. Be sure that the *large end* of the spring is on top. **Measure and record** the length of the spring with nothing attached to it. Be sure to measure from the first coil on top to the last coil on the bottom. Label this as L_{spring}.

3) Before starting, check to see that the motion detector cable is connected to DIG/Sonic #1 of the DataLogger interface box, and that the interface unit is turned on. If you are unsure, check with your TA.

4) Click on the link on the lab website to open LoggerPro. You should see three graphs – *x* vs. *t*, *v* vs. *t*, and *a* vs. *t*.

5) Position the motion detector on the floor directly under the spring. Do this by sighting through the spring from above to locate the appropriate position of the detector on the floor. This is important because the detector needs to "see" the mass you will hang throughout the motion.

6) Attach the mass holder to the bottom end of the spring and add a 100-g mass to the mass holder.

7) One partner should operate the computer and the other should pull the mass downwards about 10 cm.

8) As one partner releases the mass (do not push it – just let it go), the other should hit the *green* button on the top-center of the screen in LoggerPro (each time you hit the *green* button, the pervious plots are erased and new ones are created). Verify that the graphs appear similar to sine or cosine curves, so that the detector is "seeing" the object clearly. You can stop the data collection by hitting the *red* button (where the *green* button was).

9) Take the time now to adjust the axes of any of the graphs so the data appear clearly on each graph. This can be accomplished by double-clicking on any of the graphs and adjusting the max or min range for the vertical axis. Click on "Axes Options". You should adjust the axes so the data fills each graph as much as possible, but is still visible.

Upon completion of step 9, you should be calibrated. BE CAREFUL not to bump the detector or the table. If you do, realignment will likely be required.

Recall that when the system is in equilibrium, the gravitational force on the mass will be equal to the spring restoring force. We can use this fact to calculate the value of the spring constant later, using the following set of data:

10) You should currently have the mass holder on the spring with a 100-g mass on its base. **Record** the current total mass (mass plus the holder) and label it as m_1.

11) Be sure that the spring-mass system is in equilibrium and not moving. When it is, **measure and record** the length of the spring, consistent with the way you measured in Step 2. Label this as L_1.

12) Place a 50-g mass on the mass holder, adding it to the 100-g mass already there. **Record** the new total mass (mass plus the holder) and label it as m_2.

13) Be sure that the spring-mass system is in equilibrium. When it is, **measure and record** the length of the spring. Label this as L_2.

14) Place another 50-g mass on the mass holder. **Record** the new total mass (mass plus the holder) and label it as m_3.

15) Be sure that the spring-mass system is in equilibrium. When it is, **measure and record** the length of the spring. Label this as L_3.

Now, we will create the graphs for the oscillation of this spring-mass system. From these, we can test for the four expected behaviors of this motion (see the Lab Introduction), measure the amplitude of the motion, and measure the period of the motion.

16) Again, have one partner operate the computer and the other pull the mass. Pull the mass downwards about 10 cm.

17) **Create** your three graphs for analysis. As one partner releases the mass (do not push it – just let it go), the other should hit the *green* button on the top-center of the screen in LoggerPro. Allow the data collection to run for several seconds so that you get a decent number of cycles recorded (at least four). When you are ready to stop the data collection, hit the *red* button.

18) Be sure to adjust the axes again, if necessary, so that the data fill each window without being clipped. Also **check and verify** that the four expected behaviors (see the Lab Introduction) are evident in your data. If they are not, it is possible that the detector “lost” the mass briefly, or another significant source of error has interfered. Create a new set of graphs in that case. When

you are happy with the appearance of your graphs, **Print** a copy for each partner. Label your graphs with “200 g” to note the additional mass that was on the holder when you made these graphs.

19) Remove two 50-g masses so the mass holder contains only 100 g of additional mass. Switch partner positions (the mass operator should now operate the computer, and *vice versa*) and repeat steps 16-18 to produce another set of plots. Be sure to label the plots made with “100 g” versus the “200 g”, so you don’t confuse them with the plots you created the first time.

As always, be sure to organize your data records for presentation in your lab report, using tables and labels where appropriate.

Data Analysis

Consider the stretched spring lengths L_1, L_2, and L_3. Compute the elongation of the spring in each case: $x_i = L_i - L_{\text{spring}}$, where i = 1, 2, and 3.

In each case, there was an associated mass hanging on the spring, m_1, m_2, or m_3. Using the mass for each case and the amount of stretch x you have calculated, find a value for the spring constant in each case. Recall from the introduction that in equilibrium, $|F_{\text{spring}}| = |F_{\text{gravity}}|$, or $kx = mg$ (g = 9.8 m/s^2) . Label each of your results as k_1, k_2, and k_3.

Average your results for k and label this as k_{avg}. This is the value of k we will use for the spring-mass system for all further calculations.

Examine your graph for the *position* vs. *time* when there was 200 g of additional mass on the mass holder. Use the graph to determine the amplitude, A, and record your result. Consider Figure 9.1 for aid in thinking about the measurement. The amplitude is the greatest distance from the equilibrium position the object had during the motion.

Question 1: Is your amplitude close to 10 cm? Why might we expect this to be about 10 cm?

Examine each of your graphs for when there was 200 g of additional mass on the mass holder and, again, verify that the four expected behaviors of the motion are represented.

Question 2: Identify examples of moments in time from your graphs when each of the four behaviors are evident (these will not all happen at the same time, but a couple might!). Mark these moments in time on your graphs using a “ • ” along each curve. In answering this question, quote the relevant times you have chosen, describe what behaviors are present at each time, and explain why your results do or do not make sense. What is the spring-mass system doing at these moments in time?

Note that when you made these graphs the additional mass was 200 g. Also, we are using k_{avg} as our value for k, and the value for x in any of our equations is the displacement of the mass from the equilibrium position.

Choose a moment in time when the object is at a maximum displacement. At this moment $x = A$. What is v at this moment? Calculate the total energy at this moment using Eq. 2. Label this energy as E_1.

Choose another moment in time when the object is moving at a maximum velocity. What is the displacement of the object from the equilibrium position at this time? Is it zero like it should be? Calculate the total energy at this moment using Eq. 2. Label this energy as E_2.

Choose another moment in time when the object is neither at its maximum position nor its maximum velocity. What is the velocity at this moment? What is the displacement of the object from the equilibrium position at this time? Calculate the total energy at this moment using Eq. 2. Label this energy as E_3. Later we will evaluate these conservation of energy calculations.

Examine your graph for the *position* vs. *time* again, when there was 200 g of additional mass on the mass holder. Use the graph to determine the period, T, and record your result. Recall that the period is the time it takes for the object to go through one complete cycle of its motion. This is represented by the time between peaks on the *position* vs. *time* graph.

Calculate the frequency of the motion using your period and Eq. 3. Label this as $f_{actual200}$. Note that the frequency will have units of 1/s, often called Hertz (Hz).

Now, use Eq. 4, the k_{avg} you calculated, and the mass of the object when you made your graphs (should have been m_3) to calculate the predicted frequency. Label your result as $f_{predict200}$.

Finally, consider the graphs that you made with 100 g of additional mass on the mass holder. From these graphs, determine the period of the oscillation, and calculate its frequency using this period and Eq. 3. Label this as $f_{actual100}$.

Question 3: Is the amplitude of the position graph with the 100 g on the mass holder similar to that on the position graph using 200 g on the mass holder? Should it be? Explain why or why not. Then, compare the value of the frequencies you calculated in the two cases. Are they the same? Why or why not? Consider Eq. 4 when answering.

Error Analysis

Consider the total energies you calculated (E_1, E_2, and E_3). Find the percent difference between each of these energies. You should have three results here – one for each pair of energies. The percent difference between any of the two energies is given by:

$$\%\text{diff}_{ij} = \frac{\left|E_i - E_j\right|}{\left[(E_i + E_j)/2\right]} \times 100\%$$

This is very similar to percent error except we are dividing by the average of the two quantities since we do not have an "accepted" value for comparison.

Question 4: Was energy conserved during the motion? Explain your conclusion based on your data.

Consider your results for the frequencies you found, $f_{\text{actual200}}$ and $f_{\text{predict200}}$. Find the percent error of the measured frequency $f_{\text{actual200}}$ compared to the expected frequency $f_{\text{predict200}}$.

Question 5: Remember that we found $f_{\text{predict200}}$ from Eq. 4. Comment on the validity of Eq. 4, given your measurements and comparison and explain your conclusion.

Questions and Conclusions

Be sure to address Questions 1-5 and describe what has been verified and tested by this experiment. What are the likely sources of error? Where might the physics principles investigated in this lab manifest in everyday life, or in a job setting?

Pre-Lab Questions

Please read through all the instructions for this experiment to acquaint yourself with the experimental setup and procedures, and develop any questions you may want to discuss with your lab partner or TA before you begin. Then answer the following questions and type your answers into the Canvas quiz tool for "The Spring-Mass Oscillator," and submit it before the start of your lab section on the day this experiment is to be run.

PL-1) A spring that hangs vertically is 25 cm long when no weight is attached to its lower end. Steve adds 250 g of mass to the end of the spring, which stretches to a new length of 37 cm. What is the spring constant, k, in N/m?

PL-2) Students performing this experiment use Eq. 4 to calculate the frequency of oscillation of their mass to be 0.65 s^{-1} (that is, 0.65 Hz). Predict the time, in seconds, between successive peaks in the *position* vs. *time* plot they should expect to obtain when they measure the oscillation.

A mass and holder with a total mass of 350 g is hung at the lower end of a spring with a spring constant k of 53.0 N/m. The mass is pulled down 7.0 cm below the equilibrium point and released, setting the mass-spring system into simple harmonic. *[Use these data to answer questions PL-3 through PL-5]*.

PL-3) What is the frequency of this motion in Hertz?

PL-4) What is the total mechanical energy in the spring-mass system, in Joules, at the moment it is released?

PL-5) After the mass is released, its position and velocity change as the potential energy of the system is converted into the kinetic energy of the mass. At some point, all of the mechanical energy is in the form of kinetic energy (the mass has its maximum velocity), and the potential energy of the spring-mass is zero. Now, imagine you stopped the mass, then restarted the oscillation by pulling the mass 9.0 cm below the equilibrium point. The maximum velocity the mass obtains will be
(A) larger, because more potential energy is stored in the system so more kinetic energy results.
(B) larger, because the velocity of the initial pull adds to the second pull.
(C) smaller, because more potential energy is stored in the system so less kinetic energy results.
(D) smaller, because the mass starts at a lower position, so its peak velocity will be lower.
(E) the same, because energy is conserved.